A Comparison of Cowell's Method and a Variation-of-Parameters Method for the Computation of Precision Satellite Orbits: Addendum 1

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Additional test cases using a precision special perturbations program employing either Cowell's method or a variation-of-parameters method to compute an elliptical orbit are analyzed to determine which method is more efficient. The results obtained indicate that the variation-of-parameters method with a predict-only integrator and Cowell's method with a predict-partial-correct integrator are equally efficient and both are significantly more efficient than Cowell's method with a predict-correct integrator. Either of these two methods for computing precision satellite orbits offers the potential for reducing the total costs of computations during orbit design and computer execution time during real-time mission operations for future orbiter projects.

I. Introduction

The primary objective of the first phase of this study was to determine the best mode of integration to be used with the variation-of-parameters method when computing precision satellite orbits. Reference 1 shows that the best mode of integration of those tried is the predict-only, sixth-order, variable-step mode with a local error control proportional to r_a/r, where r_a is the apoapsis distance. In addition, the conclusion is reached that the variation-of-parameters method with this mode of integration should exhibit an improvement over Cowell's method of from 10 to 20% in the central processing unit (CPU) time and in the total cost (Ref. 1 or 2).

The objective of the second phase of this study, and the subject of this article, is to determine an accurate measure of the improvement, if any, to be expected from using the variation-of-parameters method in place of Cowell's method when computing precision satellite orbits.

II. Discussion

The results and experience gained in phase one of this study have led to a change in the original course of investigation. Initially, the intent was to compare the variation-of-parameters method using the mode of integration deemed best in Ref. 1 with Cowell's method using a predict-correct mode. However, at the end of phase one of this study, Cowell's method using a predict-partial-correct mode of integration appeared to be as efficient as the variation-of-parameters method.

Consequently, phase two of this study compares the following four processes of orbit prediction:

(1) The first process is the variation-of-parameters method with a predict-only, sixth-order, variable-step ($ERMX/ERMN \sim r_a/r$) integrator. In this process the eight parameter a_x, a_y, a_z, h_x, h_y, h_z, n, and L are integrated. As discussed in Ref. 1, this process requires that the probe ephemeris file (PEF) be written more frequently and accrues larger input and output costs than in the Cowell processes.

(2) The second process is also the variation-of-parameters method with a predict-only, sixth-order, variable-step ($ERMX/ERMN \sim r_a/r$) integrator. But in this process the four parameters h_x, h_y, h_z, and L, and two of the parameters a_x, a_y, and a_z (depending on the initial state vector) are integrated. In addition, the number of times the PEF is written is reduced by the technique described in Ref. 1. Thus, the input and output cost in the variation-of-parameters method becomes more comparable to the cost in Cowell's method. (Note that a similar reduction in the number of PEF records could also be made in Cowell's method.)

(3) The third process is Cowell's method with a predict-correct, tenth-order, variable-step (constant *ERMX* and *ERMN*) integrator. (This process is presently used for mission operations.)

(4) The fourth process is Cowell's method with a predict-partial-correct, tenth-order, variable-step (constant *ERMX* and *ERMN*) integrator.

In order to obtain accurate cost-versus-accuracy data, each of the four processes of orbit prediction was used to generate trajectory data in such a way that no calibration factors were necessary. For example, the special output used in phase one was eliminated and the improved algorithm for solving the modified Kepler's equation was used. The cost and accuracy criteria used in this article are the same as those used in phase one (Ref. 1), with the exception that the total dollar cost of computation has been added as an additional cost variable.

Sixteen cases were run in the second phase of this study. Each case used the same initial state vector (that of the *Mariner* '71 Mission A orbit) and one of the processes (1) to (4) of orbit prediction previously described. Four cases were run using process (1) and four cases were run using process (2). The four cases in each of the two sets differed only in the proportionality constants used in the local error control. Three cases were run using process (3) and five cases were run using process (4). The cases in each of these two sets differed only in the constant values of the local error control.

III. Results and Conclusions

Tables 1 and 2 present the cost and accuracy data for the variation-of-parameters method and Cowell's method, respectively. Figures 1 to 4 present plots of the CPU time versus ($|\Delta \mathbf{r}|$), the total cost (dollars) versus ($|\Delta \mathbf{r}|$), the core time product versus ($|\Delta \mathbf{r}|$), and the CPU time versus throughput time, respectively, for each of the four processes of orbit prediction. Based upon these tables and figures, the following conclusions are made:

(1) The variation-of-parameters method integrating six parameters is not significantly more efficient than Cowell's method with a predict-partial-correct integrator.

(2) The variation-of-parameters method integrating six parameters is slightly more efficient than the variation-of-parameters method integrating eight parameters.

(3) The variation-of-parameters method integrating six parameters and Cowell's method with a predict-partial-correct integrator are both significantly more efficient than Cowell's method with a predict-correct integrator. The CPU times are approximately 20% less and the total costs are approximately 8% less. These percentages will be even larger for perturbative functions which are more complex than the one used in this study.

(4) The core time product is not a reliable cost variable even on a dry system (Ref. 1).

(5) The correlation between the CPU time and the throughput time is not strong even on a dry system. Consequently, a smaller CPU time does not guarantee a smaller throughput time.

Conclusions (4) and (5) indicate the need for a large sample of data in a study of this type.

IV. Future Study

The third and final phase of this study will compare the variation-of-parameters method with Cowell's method in the case of a nearly circular orbit. A recommendation as to whether the variation-of-parameters method should be included in the standard production and mission operations versions of DPTRAJ will then be made based upon the three phases of this study.

References

1. Dallas, S. S., and Rinderle, E. A., "A Comparison of Cowell's Method and a Variation-of-Parameters Method for the Computation of Precision Satellite Orbits," *Section Technical Memorandum*, 392-66, Sept. 30, 1971 (JPL internal document).
2. Dallas, S. S., and Rinderle, E. A., "A Comparison of Cowell's Method and a Variation-of-Parameters Method for the Computation of Precision Satellite Orbits," in *The Deep Space Network Progress Report*, Technical Report 32-1526, Vol. V, pp. 74–78. Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1971.

Table 1. Cost versus accuracy for the variation-of-parameters method

Method	Local error control: ERMX	Local error control: ERMN	Accuracy[a]: $\lvert\Delta r\rvert$, m	Accuracy[a]: $\lvert\Delta \dot{r}\rvert$, m/s	Cost: CPU time, s	Cost: Core time product, kword hours	Cost: Total cost, dollars	Cost: Throughput time, s
1. Predict-only with eight integrals	$10^{-9}(r_a/r)$	$10^{-13}(r_a/r)$	24.98 107.54	0.0028 0.0524	340	10.64	112.32	618
2. Predict-only with eight integrals	$5 \times 10^{-9}(r_a/r)$	$5 \times 10^{-13}(r_a/r)$	227.74 1007.20	0.0255 0.4810	293	8.81	95.17	559
3. Predict-only with eight integrals	$10^{-8}(r_a/r)$	$10^{-12}(r_a/r)$	537.61 2398.12	0.0606 1.1662	273	9.25	96.51	561
4. Predict-only with eight integrals	$5/2 \times 10^{-8}(r_a/r)$	$5/2 \times 10^{-12}(r_a/r)$	1614.54 7305.38	0.1834 3.5499	255	8.68	90.84	515
5. Predict-only with six integrals	$10^{9}(r_a/r)$	$10^{-13}(r_a/r)$	22.22 95.96	0.0024 0.0467	332	10.44	110.28	642
6. Predict-only with six integrals	$5 \times 10^{-9}(r_a/r)$	$5 \times 10^{-13}(r_a/r)$	243.04 1084.83	0.0274 0.5276	287	9.51	99.63	583
7. Predict-only with six integrals	$10^{-8}(r_a/r)$	$10^{-12}(r_a/r)$	477.76 2130.36	0.0536 1.0364	270	9.16	95.69	613
8. Predict-only with six integrals	$5/2 \times 10^{-8}(r_a/r)$	$5/2 \times 10^{-12}(r_a/r)$	1681.22 7551.43	0.1906 3.6713	250	8.68	90.50	567

[a]The errors in each of the eight cases occur approximately at apoapsis ($t - t_0 = 234$ h) and periapsis ($t - t_0 = 240$ h), respectively, of revolution 20.

Table 2. Cost versus accuracy for Cowell's method

Method	Local error control: ERMX	Local error control: ERMN	Accuracy[a]: $\lvert\Delta r\rvert$, m	Accuracy[a]: $\lvert\Delta \dot{r}\rvert$, m/s	Cost: CPU time, s	Cost: Core time product, kword hours	Cost: Total cost, dollars	Cost: Throughput time, s
1. Predict-correct	10^{-10}	10^{-15}	89.83 372.24	0.0096 0.1815	352	10.00	108.95	613
2. Predict-correct	2×10^{-10}	2×10^{-15}	521.26 2261.93	0.0577 1.1008	337	9.66	105.15	555
3. Predict-correct	10^{-9}	10^{-14}	2733.42 11884.88	0.3036 5.7820	301	9.68	102.15	570
4. Predict-partial-correct	10^{-11}	10^{-16}	24.14 105.22	0.0027 0.0512	329	10.26	108.67	587
5. Predict-partial-correct	10^{-10}	10^{-15}	90.71 372.33	0.0096 0.1817	292	9.43	99.47	579
6. Predict-partial-correct	$3/2 \times 10^{-10}$	$3/2 \times 10^{-15}$	138.25 577.44	0.0150 0.2814	284	8.46	92.04	513
7. Predict-partial-correct	2×10^{-10}	2×10^{-15}	374.10 1620.47	0.0411 0.7889	283	9.35	98.13	589
8. Predict-partial-correct	10^{-9}	10^{-14}	2724.17 11876.47	0.3037 5.7765	254	8.58	90.06	511

[a]The errors in each of the eight cases occur approximately at apoapsis ($t - t_0 = 234$ h) and periapsis ($t - t_0 = 240$ h), respectively, of revolution 20.

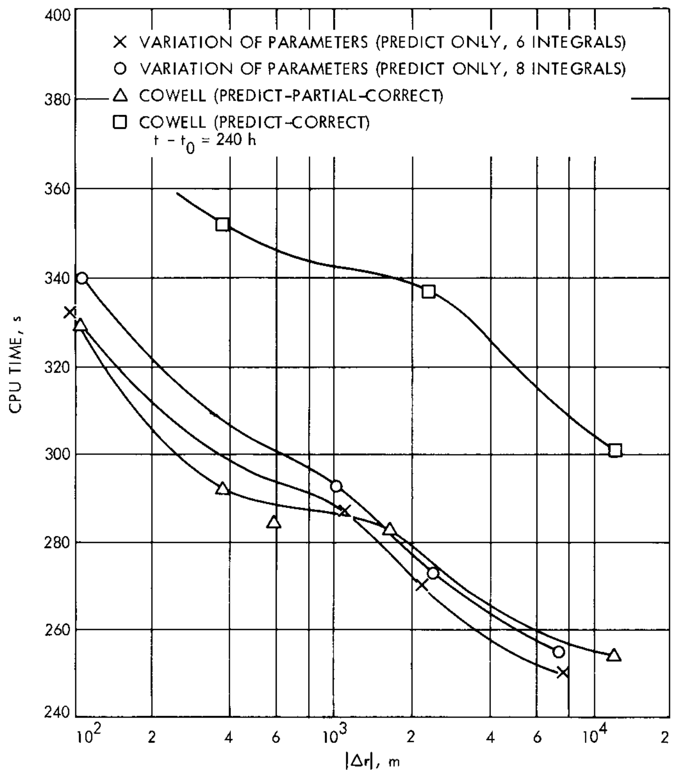

Fig. 1. Cost in CPU time versus accuracy

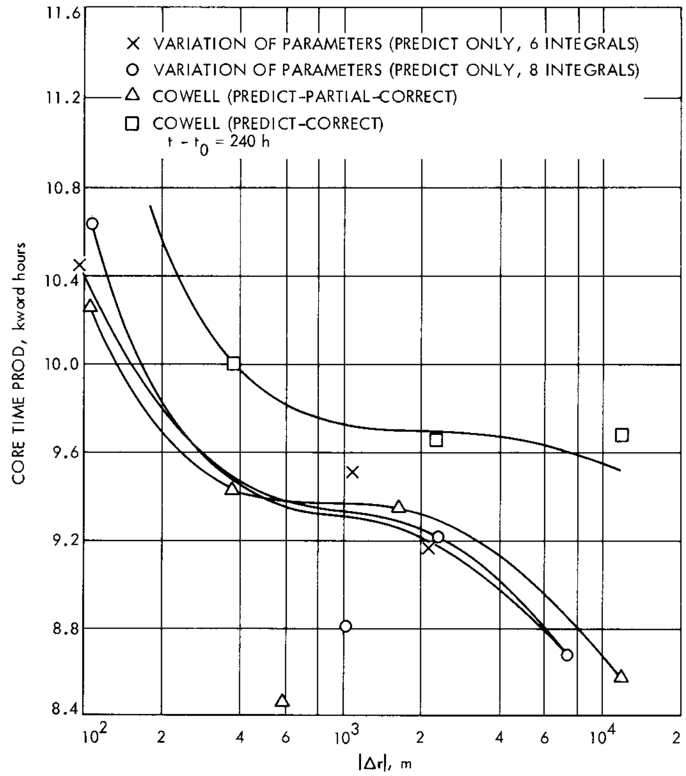

Fig. 3. Core time product versus accuracy

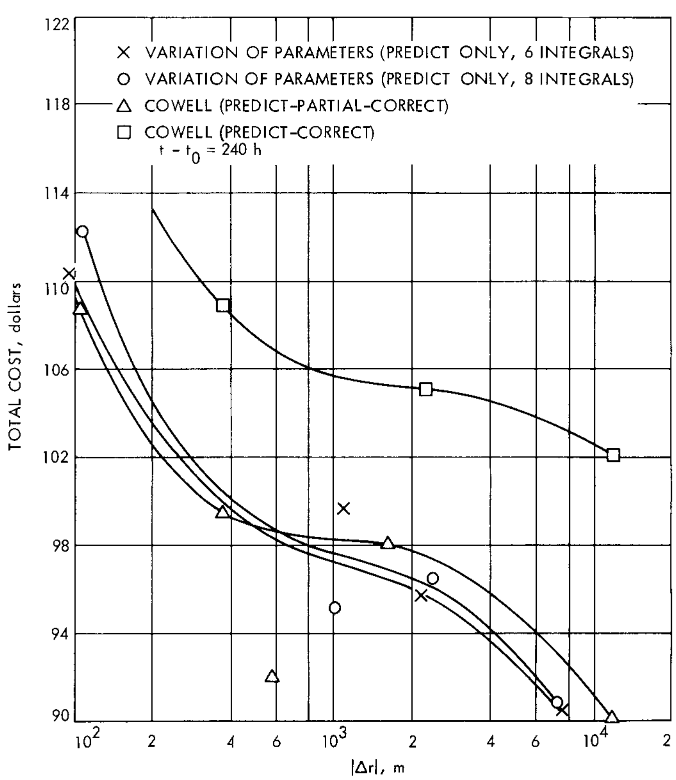

Fig. 2. Total cost versus accuracy

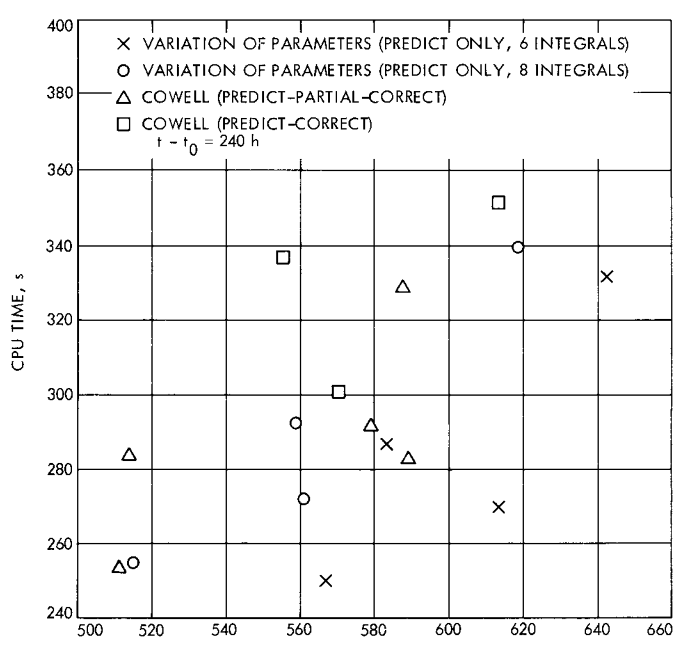

Fig. 4. CPU time versus throughput time